

**B.Sc. Semester-V Examination, 2022-23****PHYSICS [Honours]**

Course ID : 52416 Course Code : SH/PHS/503/DSE-1

Course Title : Advanced Mathematical Physics

OR

Classical Dynamics

Time : 2 Hours

Full Marks : 40

*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.***(Advanced Mathematical Physics)****UNIT-I**

1. Answer any **five** questions: 2×5=10
- Show that any tensor of rank 2 can be expressed as the sum of a symmetric and anti-symmetric tensors of rank 2.
  - Define symmetric and anti-symmetric tensor.
  - Define cyclic group.
  - If  $A_r^{pq}$  and  $B_r^{pq}$  are tensors, prove that their sum and difference are also tensors.

- Prove that Kronecker delta is a mixed tensor of rank 2.
- If  $N$  is a normal subgroup of  $G$ , show that  $xN = Nx$  for all  $x \in G$ .
- Let  $G$  be an abelian group. Prove that the subset  $H = \{g \in G : g^2 = e \text{ (identity element)}\}$  forms a subgroup of  $G$ .
- Define Riemannian space.

**UNIT-II**

2. Answer any **four** questions: 5×4=20
- Prove Bayes' theorem.
  - A covariant tensor has components  $xy, 2y-z^2, xz$  in rectangular coordinates. Find its covariant components in spherical coordinates.
  - Show that in a Cartesian coordinate system the contravariant and covariant components of a vector are identical.
  - Determine the metric tensor in (i) cylindrical and (ii) spherical coordinates.

- e) Two dice are rolled.  
 A='sum of two dice equals 3'  
 B='sum of two dice equals 7'  
 C='at least one of the dice shows a 1'
- What is  $(A|C)$ ?
  - What is  $(B|C)$ ?
  - Are A and C independent?  $2+2+1$
- f) Define homomorphism and isomorphism of a group.

### UNIT-III

3. Answer any **one** question:  $10 \times 1 = 10$
- Express the Laplacian of  $\phi$ ,  $\nabla^2\phi$ , in (i) cylindrical coordinates (ii) spherical coordinates.  $5+5=10$
  - What is binomial distribution? Find the probability distribution function of binomial distribution. Hence, find Gaussian distribution with proper condition.  $2+3+5=10$

### (Classical Dynamics)

### UNIT-I

1. Answer any **five** of the following:  $2 \times 5 = 10$
- What do you mean by pseudo force?
  - What is Gyro-radius and Gyro-frequency?
  - What is canonical transformation? What is the condition for the transformation to be canonical?
  - Differentiate streamline motion and turbulent motion of fluid.
  - What are stable and unstable equilibrium? Explain with diagram.
  - Write down the Lagrangian of a particle, moving on the surface on the sphere.
  - What do you mean by degrees of freedom? Find out the degree of freedom of a tri-atomic molecule.
  - State D'Alembert's principle and write down its mathematical form.

**UNIT-II**

2. Answer any **four** of the following: 5×4=20
- a) A bead slides without friction through a frictionless wire in the shape of a cycloid with equations  $x = a(\theta - \sin \theta)$ ,  $y = a(1 + \cos \theta)$  where  $0 \leq \theta \leq 2\pi$ . Find (i) the Lagrangian function and (ii) the equation of motion. 2+3
- b) Explain how a charged particle in an external magnetic field undergoes circular motion. 5
- c) Explain the following statement– “The Navier-Stokes equations mathematically express conservation of momentum and conservation of mass for Newtonian fluids”. 5
- d) Write down Euler-Lagrange equation. Show that the shortest path between two points in Euclidean space is basically a straight line. 1+4
- e) Obtain the Lagrange’s equation of motion for small oscillations of a system in the neighborhood of stable equilibrium. 5
- f) The potential energy as a function of the force between two atoms in a diatomic molecule is given by:  $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$ , where,  $a$  and  $b$  are positive constants and  $x$  is the distance between the atoms. Find out the stable equilibrium for the system of two atoms. 5

**UNIT-III**

3. Answer any **one** of the following: 10×1=10
- a) i) Show that the differential equation describing the motion of a particle in a central field can be written as:
- $$\frac{mh^2}{2r^4} \left[ \left( \frac{dr}{d\theta} \right)^2 + r^2 \right] - \int f(r) dr = E;$$
- where symbols have their usual meanings.
- ii) While considering equation of motion for a particle under central force, show that  $r^2\dot{\theta} = \text{constant}$ .
- iii) Derive the Lagrangian of a particle, constrained to slide down (without rolling) through a friction-less inclined plane surface of inclination- $\theta$ . 6+2+2
- b) Prove that  $x^2 + y^2 + z^2 - c^2t^2$  remains invariant under Lorentz transformation in all frame of reference. Explain the phenomenon of length contraction. Write down the postulates of special theory of relativity. 5+3+2